### INTRODUCTION

Diesel engine buses still comprise the majority of the bus fleet in the United States even with the problem of diesel exhaust and greenhouse gas emissions. Electric buses, which generate no emissions on the road, provide a promising green alternative for bus fleets. However, due to the limitation of battery technology, electric buses suffer the disadvantages of high cost, limited travel distance, and long charging time. The technique of dynamic wireless charging, which allows electric buses to charge while traveling, can effectively alleviate the drawbacks of electric buses. The key design variables of deploying dynamic wireless charging facilities for an electric bus system are the battery size and the location of the wireless charging facilities. This paper addresses the problem of selecting optimal locations for the wireless charging facilities and designing battery size for an electric bus system simultaneously. A mathematical model was developed with the objective of minimizing the total implementation cost. And the results demonstrated that the proposed model could effectively solve the optimal deployment problem of dynamic wireless charging facilities for an electric bus system.

### OPTIMIZATION MODEL

#### Constraints of Power Transmitters

\[
x_{ij} = \begin{cases} 
1 & \text{if link } (i, j) \text{ is on a power transmitter} \\
0 & \text{otherwise} 
\end{cases} \\
y_i = \begin{cases} 
1 & \text{if node } i \text{ is the starting point of power transmitter} \\
0 & \text{otherwise} 
\end{cases} \\
z_i = \begin{cases} 
1 & \text{if node } i \text{ is a merging point of a power transmitter} \\
0 & \text{otherwise} 
\end{cases} \\
y_i \leq \sum_{(m, i) \in L^+} x_{mi} & \forall i \in N \\
y_i \leq 1 - x_{mi} & \forall i \in N, \forall (m, i) \in L_i \\
y_i \geq x_{ij} - \sum_{(m, j) \in L^+} x_{mj} & \forall i \in N, \forall (i, j) \in L_i^+ \\
x_{ij} \in \{0, 1\} & \forall (i, j) \in L \\
y_i \in \{0, 1\} & \forall i \in N \\
z_i \leq \sum_{(m, i) \in L^+} x_{mi} & \forall i \in N^2 \\
z_i \geq x_{mi} & \forall i \in N^2, \forall (m, i) \in L_i^+
\]

#### Energy Power Constraints

\[e_{ki} \text{ the battery power at node } i \text{ for electric buses on line } k\]
\[c_{ki} \text{ the energy consumption on link } (i, j) \text{ for electric buses on line } k\]
\[s_{ki} \text{ the energy supply on link } (i, j) \text{ for electric buses on line } k\]

\[e_{ki} = e_{ki}^{in} + s_{ki} \quad \forall (i, j) \in L_k\]
\[e_{ki} \leq e_{ki}^{max} \quad \forall k \in K, \forall i \in N_k\]
\[e_{ki} \geq e_{ki}^{min} \quad \forall k \in K, \forall i \in N_k\]

Energy Consumption and Supply Models

\[
\begin{align*}
\eta_k &= \left(\frac{\eta_k^{in} (\sigma_{ki} w_{ij}^P f_i + \sigma_{ki} w_{ij}^P f_i^2)}{2} \right) \\
&\quad + \left(\eta_k^{out} e_{ki} + \eta_k^{out} (1 - \xi_{ki}) w_{ki} e_{ki}^o \right) + d_{ij} \left(\eta_k^{out} e_{ki} + \eta_k^{out} (1 - \xi_{ki}) w_{ki} e_{ki}^o \right) w_{ki}^o
\end{align*}
\]

Energy Supply

\[s_{kj} \leq x_{ij} p_{kj} s_{ki} \quad \forall k \in K, \forall (i, j) \in L_k\]
\[s_{kj} \geq 0 \quad \forall k \in K, \forall (i, j) \in L_k
\]

Optimization Model

\[
\min \left\{ \sum_{(i, j) \in L} y_i - \sum_{i \in N} \sum_{m \in M} x_{mi} + \sum_{k \in K} \sum_{(i, j) \in L_k} z_{ij} \right\} \\
+ a \sum_{(i, j) \in L} d_{ij} x_{ij} + a \sum_{(k, i) \in L} t_{ik} e_{ik}^{max}
\]

\[
\text{s.t.} \\
y_i \leq \sum_{(j, m) \in I} x_{ij} & \forall i \in N \\
y_i \leq 1 - x_{mi} & \forall i \in N, \forall (m, i) \in L_i \\
y_i \geq x_{ij} - \sum_{(m, j) \in L^+} x_{mj} & \forall i \in N, \forall (i, j) \in L_i^+ \\
z_i \leq \sum_{(m, i) \in L^+} x_{mi} & \forall i \in N^2 \\
z_i \geq x_{mi} & \forall i \in N^2, \forall (m, i) \in L_i^+
\]

### NUMERICAL STUDY

The shuttle system of Utah State University

#### Comparison with Stationary Charging

**Battery Size Comparison**

<table>
<thead>
<tr>
<th>Shuttle Line</th>
<th>Stationary Charging (kWh)</th>
<th>Dynamic Charging (kWh)</th>
<th>Battery Size Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red (#1)</td>
<td>97.7</td>
<td>55.6</td>
<td>43.1%</td>
</tr>
<tr>
<td>Green (#2)</td>
<td>34.5</td>
<td>21.8</td>
<td>39.4%</td>
</tr>
<tr>
<td>Blue (#3)</td>
<td>47.5</td>
<td>32.5</td>
<td>31.6%</td>
</tr>
<tr>
<td>Purple (#4)</td>
<td>86.4</td>
<td>45.2</td>
<td>47.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total cost comparison</th>
<th>Cost($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Track cost</td>
<td>870,000</td>
</tr>
<tr>
<td>Total</td>
<td>3,432,497</td>
</tr>
</tbody>
</table>

### CONCLUSION

The dynamic charging technique provides an attractive alternative for charging electric buses. This study demonstrated that it offers substantial savings over the traditional stationary charging system, and hence, it is expected to be widely adopted in the future. The proposed model provides practitioners an effective tool to determine the optimal deployment of power transmitters and battery size of each bus line.